Appendix

Map Scale and Projections

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Unaided, our human senses provide a limited view of our surroundings. To overcome those limitations, humankind has developed powerful vehicles of thought and communication, such as language, mathematics, and graphics. Each of those tools is based on elaborate rules; each has an information bias, and each may distort its message, often in subtle ways. Consequently, to use those aids effectively, we must understand their rules, biases, and distortions. The same is true for the special form of graphics we call maps: we must master the logic behind the mapping process before we can use maps effectively.

A fundamental issue in cartography, the science and art of making maps, is the vast difference between the size and geometry of what is being mapped—the real world, we will call it—and that of the map itself. Scale and projection are the basic cartographic concepts that help us understand that difference and its effects.

▲ FIGURE A-1 Common expressions of map scale.

Map Scale

Our senses are dwarfed by the immensity of our planet; we can sense directly only our local surroundings. Thus, we cannot possibly look at our whole state or country at one time, even though we may be able to see the entire street where we live. Cartography helps us expand what we can see at one time by letting us view the scene from some distant vantage point. The greater the imaginary distance between that position and the object of our observation, the larger the area the map can cover, but the smaller the features will appear on the map. That reduction is defined by the *map scale*, the ratio of the distance on the map to the distance on the earth. Map users need to know about map scale for two reasons: so that they can convert measurements on a map into meaningful real-world measures and so that they can know how abstract the cartographic representation is.

REAL-WORLD MEASURES. A map can provide a useful substitute for the real world for many analytical purposes. With the scale of a map, for instance, we can compute the actual size of its features (length, area, and volume). Such calculations are helped by three expressions of a map scale: a word statement, a graphic scale, and a representative fraction.

A *word statement* of a map scale compares X units on the map to Y units on the earth, often abbreviated "X unit to Y units." For example, the expression "1 inch to 10 miles" means that 1 inch on the map represents 10 miles on the earth (Figure A-1). Because the map is always smaller than the area that has been mapped, the ground unit is always the larger number. Both units are expressed in meaningful terms, such as inches or centimeters and miles or kilometers. Word statements are not intended for precise calculations but give the map user a rough idea of size and distance.

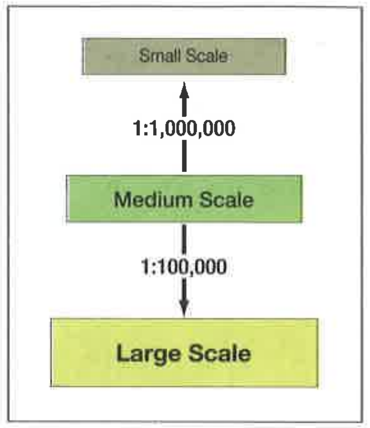
A *graphic scale,* such as a bar graph, is concrete and therefore overcomes the need to visualize inches and miles that is associated with a word statement of scale (see Figure A-1). A graphic scale permits direct visual comparison of feature sizes and the distances between features. No ruler is required; any measuring aid will do. It needs only be compared with the scaled bar; if the length of 1 toothpick is equal to 2 miles on the ground and the map distance equals the length of 4 toothpicks, then the ground distance is 4 times 2, or 8 miles. Graphic scales are especially convenient in this age of copying machines, when we are more likely to be working with a copy than with the original map. If a map is reduced or enlarged as it is copied, the graphic scale will change in proportion to the change in the size of the map and thus will remain accurate.

The third form of a map scale is the *representative fraction* (RF). An RF defines the ratio between the distance on the map and the distance on the earth in fractional terms, such as 1/633,600 (also written 1:633,600). The numerator of the fraction always refers to the distance on the map, and the denominator always refers to the distance on the earth. No units of measurement are given, but both numbers must be expressed in the same units. Because map distances are extremely small relative to the size of the earth, it makes sense to use small units, such as inches or centimeters. Thus the RF 1:633,600 might be read as "1 inch on the map to 633,600 inches on the earth."

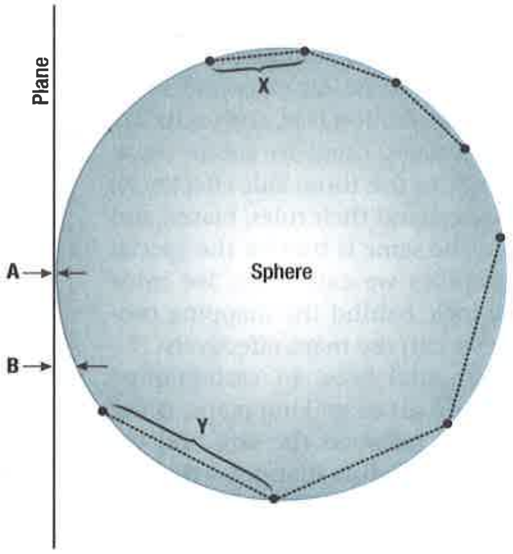
Herein lies a problem with the RF. Meaningful map-distance units imply a denominator so large that it is impossible to visualize. Thus, in practice, reading the map scale involves an additional step of converting the denominator to a meaningful ground measure, such as miles or kilometers. The unwieldy 633,600 becomes the more manageable 10 miles when divided by the number of inches in a mile (63,360).

On the plus side, the RF is good for calculations. In particular, the ground distance between points can be easily determined from a map with an RF. One simply multiplies the distance between the points on the map by the denominator of the RF. Thus a distance of 5 inches on a map with an RF of 1/126,720 would signify a ground distance of 5 X 126,720, which equals 633,600. Because all units are inches and there are 63,360 inches in a mile, the ground distance is 633,600/ 63,360, or 10 miles. Computation of area is equally straightforward with an RF. Computer manipulation and analysis of maps is based on the RF form of map scale.

GUIDES TO GENERALIZATION. Scales also help map users visualize the nature of the symbolic relation between the map and the real world. It is convenient here to think of maps as falling into three broad scale categories (Figure A-2). (Do not be confused by the use of the words large AND small in this context; just remember that the larger the denominator, the smaller the scale ratio and the larger the area that is shown on the map.) Scale ratios greater than 1:100,000, such as the 1:24,000 scale of U.S. Geological Survey topographic quadrangles, are large-scale maps. Although those maps can cover only a local area, they can be drawn to rather rigid standards of accuracy. Thus they are useful for a wide range of applications that require detailed and accurate maps, including zoning, navigation, and construction.



▲ FIGURE A-2 The scale gradient can be divided into three broad categories.



▲ FIGURE A-3 Relationships between surfaces on the round earth and a flat map.

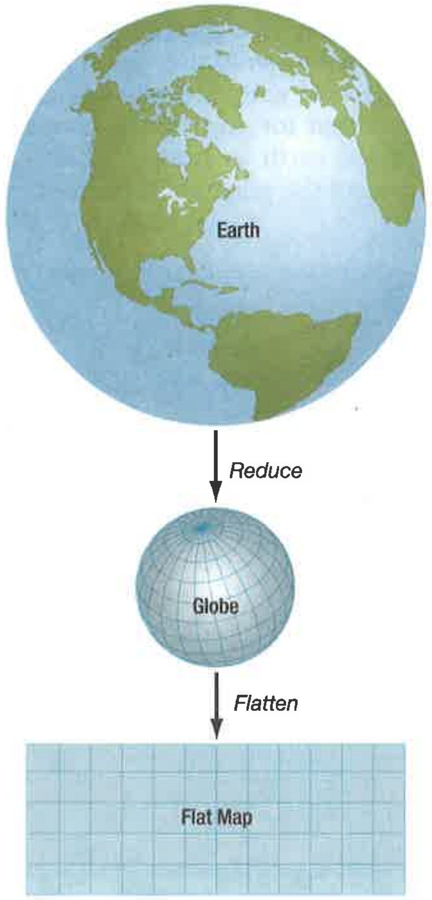
At the other extreme are maps with scale ratios of less than 1:1,000,000, such as maps of the world that are found in atlases. Those are small-scale maps. Because they cover large areas, the symbols on them must be highly abstract. They are therefore best suited to general reference or planning, when detail is not important. Medium- or intermediate-scale maps have scales between 1:100,000 and 1:1,000,000. They are good for regional reference and planning purposes.

Another important aspect of map scale is to give us some notion of geometric accuracy; the greater the expanse of the real world shown on a map, the less accurate the geometry of that map is. Figure A-3 shows why. If a curve is represented by straight line segments, short segments (*X*) are more similar to the curve than are long segments (*Y*). Similarly, if a plane is placed in contact with a sphere, the difference between the two surfaces is slight where they touch (*A*) but grows rapidly with increasing distance from the point of contact (*B*). In view of the large diameter and slight local curvature of the earth, distances will be well represented on large-scale maps (those with small denominators) but will be increasingly poorly represented at smaller scales. This close relationship between map scale and map geometry brings us to the topic of map projections.

Map Projections

The spherical surface of the earth is shown on flat maps by means of map projections. The process of "flattening" the earth is essentially a problem in geometry that has captured the attention of the best mathematical minds for centuries. Yet no one has ever found a perfect solution; there is no known way to avoid spatial distortion of one kind or another. Many map projections have been devised, but only a few have become standard. Because a single flat map cannot preserve all aspects of the earth's surface geometry, a mapmaker must be careful to match the projection with the task at hand. To map something that involves distance, for example, a projection should be used in which distance is not distorted. In addition, a map user should be able to recognize which aspects of a map's geometry are accurate and which are distortions caused by a particular projection process. Fortunately, that objective is not too difficult to achieve.

It is helpful to think of the creation of a projection as a twostep process (Figure A-4). First, the immense earth is reduced to a small globe with a scale equal to that of the desired flat map. All spatial properties on the globe are true to those on the earth. Second, the globe is flattened. Since that cannot be done without distortion, it is accomplished in such a way that the resulting map exhibits certain desirable spatial properties.



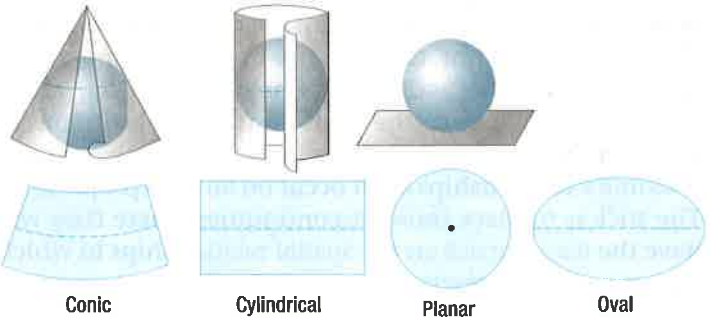
▲ **FIGURE A-4** The two-step process of creating a projection.

PERSPECTIVE MODELS. Early map projections were sometimes created with the aid of perspective methods, but that has changed. In the modern electronic age, projections are normally developed by strictly mathematical means and are plotted out or displayed on computer-driven graphics devices. The concept of perspective is still useful in visualizing what map projections do, however. Thus projection methods are often illustrated by using strategically located light sources to cast shadows on a projection surface from a latitude/longitude net inscribed on a transparent globe.

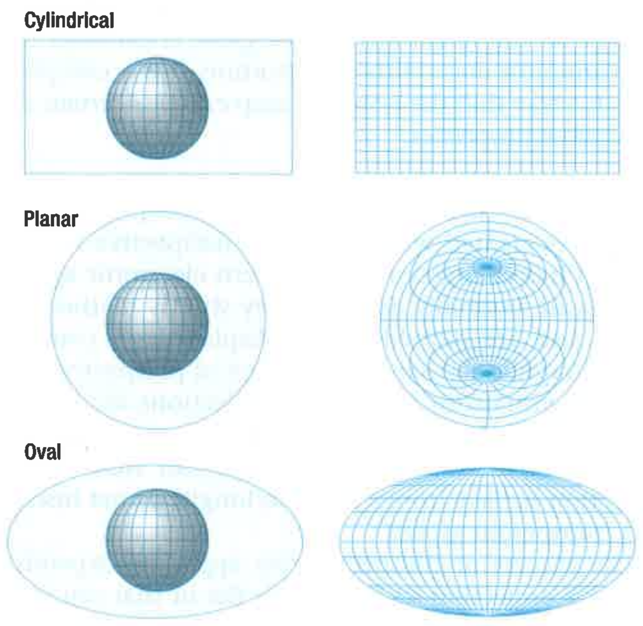
The success of the perspective approach depends on finding a projection surface that is flat or that can be flattened without distortion. The cone, cylinder, and plane possess those attributes and serve as models for three general classes of map projections: *conic, cylindrical,* and *planar* (or azimuthal). Figure A-5 shows those three classes, as well as a fourth, a false cylindrical class with an oval shape. Although the oval class is not of perspective origin, it appears to combine properties of the cylindrical and planar classes (Figure A-6).

The relationship between the projection surface and the model at the point or line of contact is critical because distortion of spatial properties on the projection is symmetrical about, and increases with distance from, that point or line. That condition is illustrated for the cylindrical and planar classes of projections in Figure A-7. If the point or line of contact is changed to some other position on the globe, the distortion pattern will be recentered on the new position but will retain the same symmetrical form. Thus centering a projection on the area of interest on the earth's surface can minimize the effects of projection distortion. And recognizing the general projection shape, associating it with a perspective model, and recalling the characteristic distortion pattern will provide the information necessary to compensate for projection distortion.

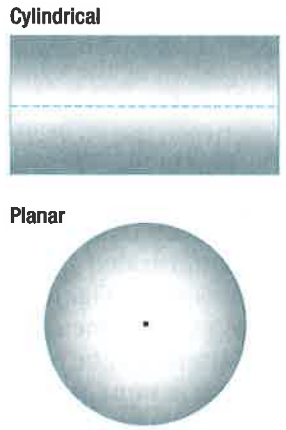
PRESERVED PROPERTIES. For a map projection to truthfully depict the geometry of the earth's surface, it



▲ **FIGURE A-5** General classes of map projections.



▲ **FIGURE A-6** The visual properties of cylindrical and planar projections combined in oval projections.



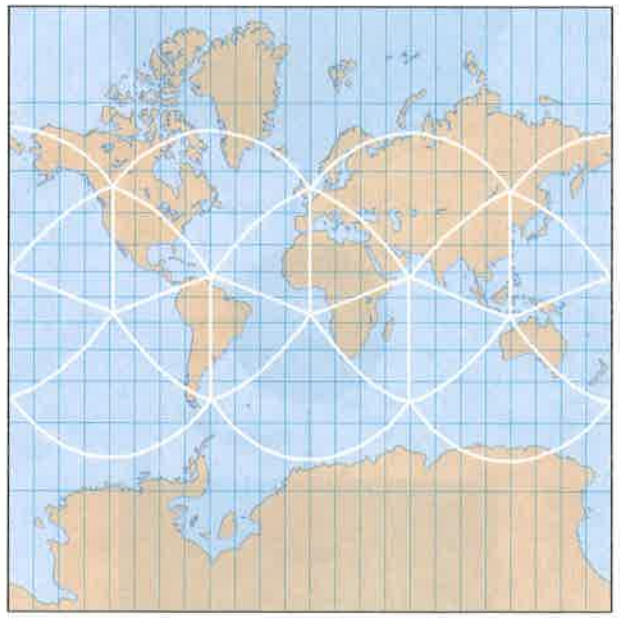
▲ **FIGURE A-7** Characteristic patterns of distortion for two projection classes. Here, darker shading implies greater distortion.

would have to preserve the spatial attributes of *distance, direction, area, shape*, and *proximity*. That task can be readily accomplished on a globe, but it is not possible on a flat map. To preserve area, for example, a mapmaker must stretch or shear shapes; thus area and shape cannot be preserved on the same map. To depict both direction and distance from a point, area must be distorted. Similarly, to preserve area as well as direction from a point, distance has to be distorted. Because the earth's surface is continuous in all directions from every point, discontinuities that violate proximity relationships must occur on all map projections. The trick is to place those discontinuities where they will have the least impact on the spatial relationships in which the map user is interested.

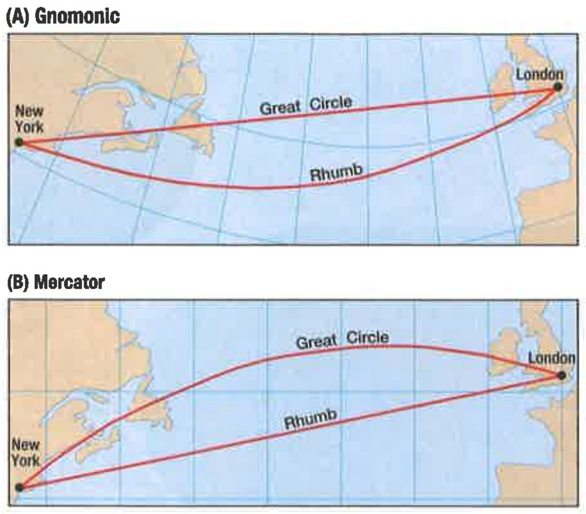
We must be careful when we use spatial terms, because the properties they refer to can be confusing. The geometry of the familiar plane is very different from that of a sphere; yet when we refer to a flap map, we are in fact making reference to the spherical earth that was mapped. A shape-preserving projection, for example, is truthful to local shapes—such as the rightangle crossing of latitude and longitude lines—but does not preserve shapes at continental or global levels. A distance-preserving projection can preserve that property from one point on the map in all directions or from a number of points in several directions, but distance cannot be preserved in the general sense that area can be preserved. Direction can also be generally preserved from a single point or in several directions from a number of points but not from all points simultaneously. Thus a shape-, distance-, or direction-preserving projection is truthful to those properties only in part.

Partial truths are not the only consequence of transforming a sphere into a flat surface. Some projections exploit that transformation by expressing traits that are of considerable value for specific applications. One of those is the famous shape-preserving *Mercator projection* (Figure A-8). That cylindrical projection was derived mathematically in the 1500s so that a compass bearing (called rhumb lines) between any two points on the earth would plot as straight lines on the map. That trait let navigators plan, plot, and follow courses between origin and destination, but it was achieved at the expense of extreme areal distortion toward the margins of the projection (see Antarctica in Figure A-8). Although the Mercator projection is admirably suited for its intended purpose, its widespread but inappropriate use for nonnavigational purposes has drawn a great deal of criticism.

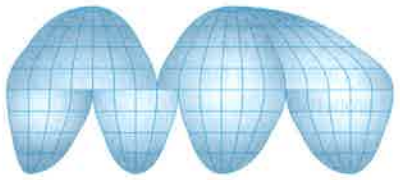
The *gnomonic projection* is also useful for navigation. It is a planar projection with the valuable characteristic of showing the shortest (or great circle) route between any two points on the earth as straight lines. Long-distance navigators first plot the great circle course between origin



▲ **FIGURE A-8** The useful Mercator projection, showing extreme area distortion in the higher latitudes.



▲ **FIGURE A-9** A gnomonic projection (A) and a Mercator projection (B), both of value to long-distance navigators.

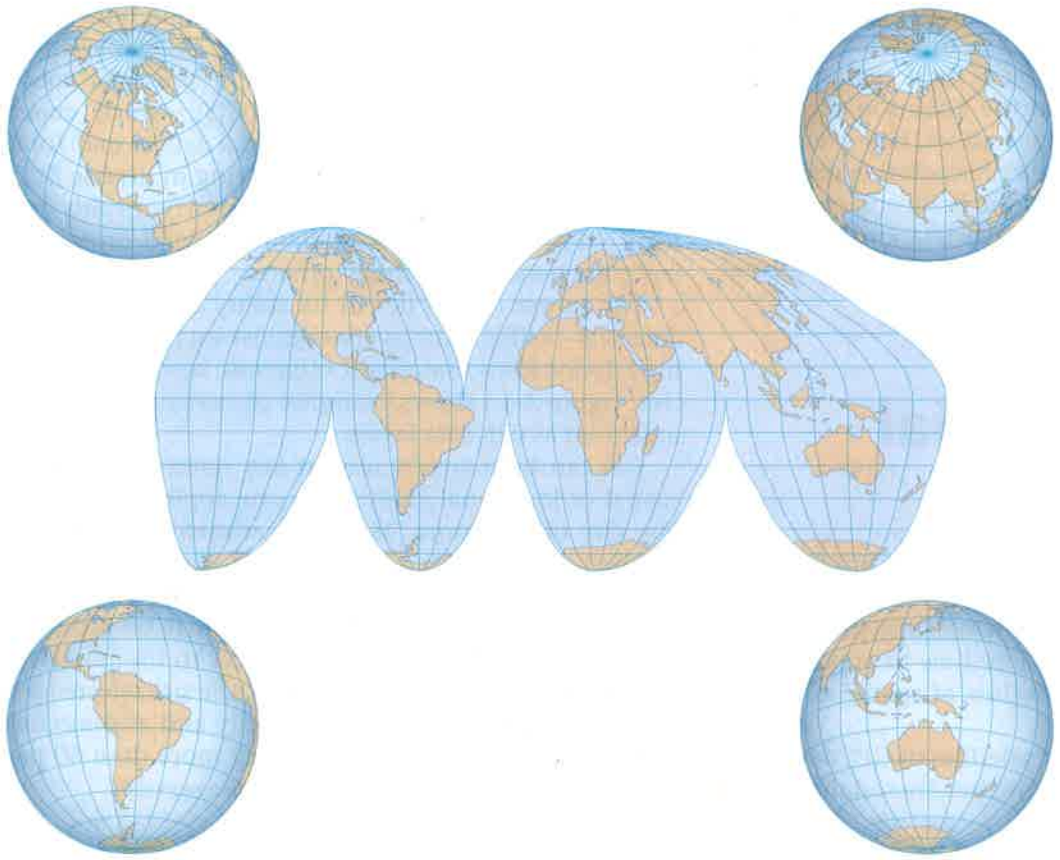


▲ **FIGURE A-11** The distortion pattern of the interrupted Goode's homolosine projection, which mimics that of cylindrical projections.

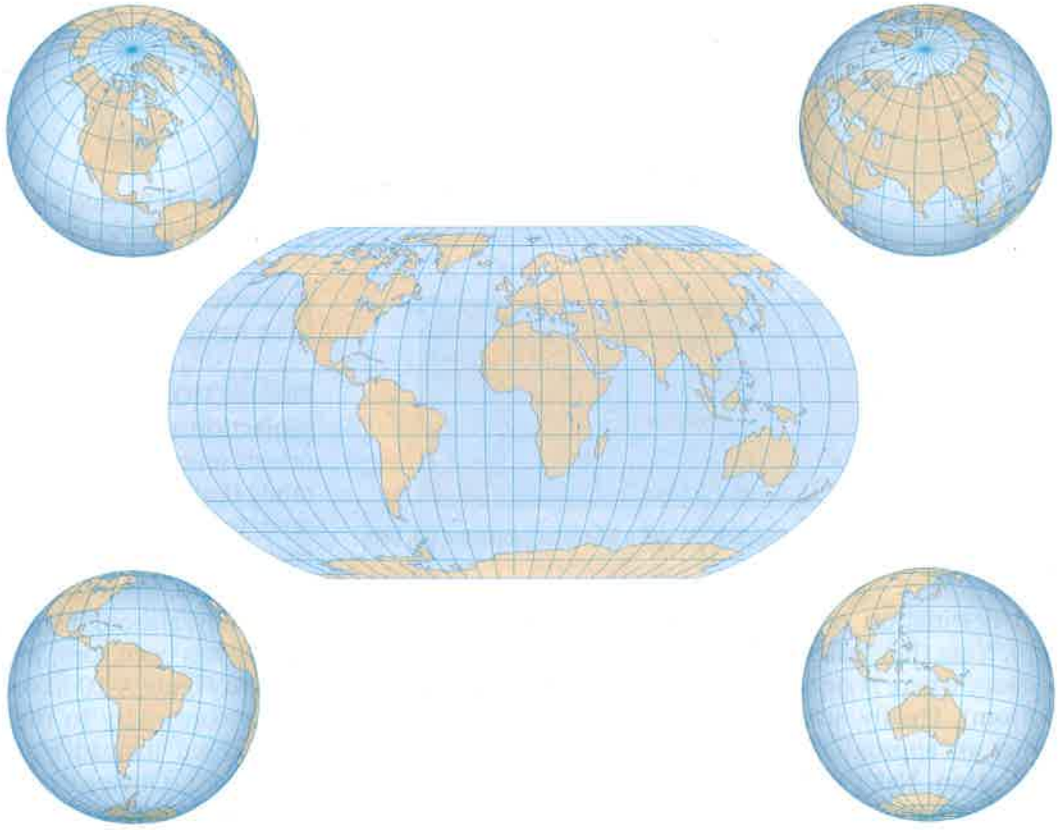
and destination on a gnomonic projection (Figure A-9, top). Next they transfer the straight line to a Mercator projection, where it normally appears as a curve (Figure A-9, bottom). Finally, using straight-line segments, they construct an approximation of that course on the Mercator projection. Navigating the shortest course between origin and destination then involves following the straight segments of the course and making directional corrections between segments. Like the Mercator projection, the specialized gnomonic projection distorts other spatial properties so severely that it should not be used for any purpose other than navigation or communications.

PROJECTIONS USED IN TEXTBOOKS. Although a map projection cannot be free of distortion, it can represent one or several spatial properties of the earth's surface accurately if other properties are sacrificed. The two projections used for world maps throughout this textbook illustrate that point well. *Goode's homolosine projection*, shown in Figure A-10, belongs to the oval category and shows area accurately, although it gives the impression that the earth's surface has been torn, peeled, and flattened. The interruptions in Figure A-10 have been placed in the major oceans, giving continuity to the land masses. Ocean areas could be featured instead by placing the interruptions in the continents. Obviously, that type of interrupted projection severely distorts proximity relationships. Consequently, in different locations the properties of distance, direction, and shape are also distorted to varying degrees. The distortion pattern mimics that of cylindrical projections, with the equatorial zone the most faithfully represented (Figure A-11).

An alternative to special-property projections such as the equal-area Goode's homolosine is the compromise projection. In that case no special property is achieved at the expense of others, and distortion is rather evenly distributed among the various properties, instead of being focused on one or several properties. The *Robinson projection*, which is also used in this textbook, falls into that category (Figure A-12). Its oval projection has a global feel, somewhat like that of Goode's homolosine. But the Robinson projection shows the North Pole and the South Pole as lines that are slightly more than half the length of the equator, thus exaggerating distances and areas near the poles. Areas look larger than they really are in the high



▲ **FIGURE A-10** An interrupted Goode's homolosine, an equal-area projection.



▶ **FIGURE A-12** The compromise Robinson projection, which avoids the interruptions of Goode's homolosine but preserves no special properties. (*Courtesy of ACSM*)

latitudes (near the poles) and smaller than they really are in the low latitudes (near the equator). In addition, not all latitude and longitude lines intersect at right angles, as they do on the earth, so we know that the Robinson projection does not preserve direction or shape either. However, it has fewer interruptions than the Goode's homolosine does, so it preserves proximity better. Overall, the Robinson projection does a good job of representing spatial relationships, especially in the low to middle latitudes and along the central meridian.

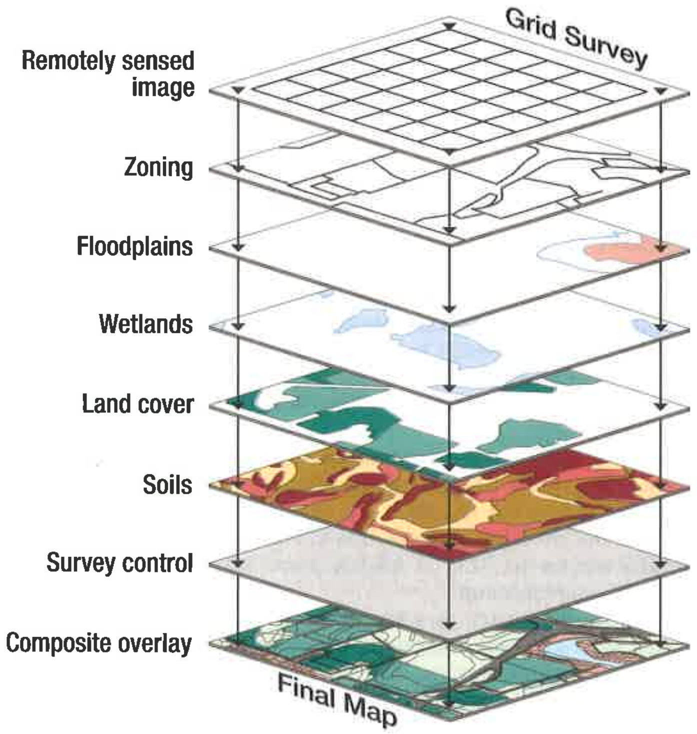
GIS and Geospatial Technologies

Today, user-friendly mapping software enables anyone with a computer to produce maps at a range of scales using a variety of projections. But a challenge remains to geographers and other users of spatial information: how to organize and present in map form the vast amounts of spatial data that are now available.

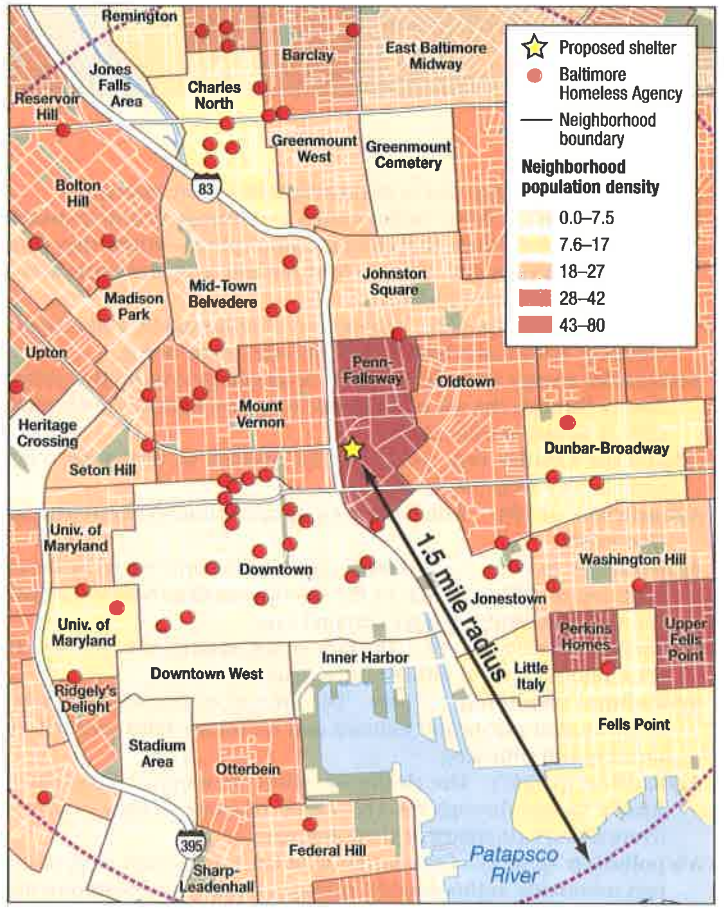
From data gathered by orbiting satellites or GPS devices to statistical data linked to spatial coordinates, these data provide a more detailed view of Earth's physical and human systems than has ever before been possible. To manage these data, geographers have developed a powerful tool—geographic information systems (GIS), which enable users to manipulate and display spatial data in map form. GIS maps contribute to problem solving in diverse fields such as science and engineering, industry, health care, retail sales, urban planning, environmental protection, law enforcement, and many others.

The power of GIS lies in its ability to map different data sets—called data layers—against each other, revealing relationships that might otherwise be difficult to detect. Figure A-13 shows examples of environmental data organized as GIS data layers.

GIS can help answer almost any question involving spatial or locational analysis. In one application of GIS, The city of Baltimore, Maryland, wanted to determine the best location for an emergency shelter for homeless people. Among many other factors considered, one main criterion for the shelter was that it be accessible from other facilities providing services to the homeless. As shown in the map in Figure A-14, the site selected was in a densely populated part of the city and at the center of a 1.5-mile-radius circle containing more than 60 percent of the city's providers of homeless services.



▲ FIGURE A-13 Within a GIS, environmental data attached to a common terrestrial reference system, such as latitude/longitude, can be stacked in layers for spatial comparison and analysis.



▲ FIGURE A-14 This GIS map shows that Baltimore's proposed homeless shelter would be near a cluster of service providers. The map's data layers include: neighborhood borders, population density (by neighborhood), and locations of homeless agencies.